International Online Math Challenge

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Past Papers 2022

Category 5

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Category-5 IMC 2022

1. Calculate the expression $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{99^2}\right)\left(1-\frac{1}{100^2}\right)$.

A) $\frac{101}{200}$ B) $\frac{100}{200}$ C) $\frac{99}{100}$ D) $\frac{99}{200}$

2. If $9^{1-a}=16$, then find 27^a.

A) $\frac{125}{64}$ B) $\frac{27}{16}$ C) $\frac{27}{64}$ D) $\frac{64}{125}$

3. Given the sequence with general term $a_n=5^n\times n!$, then find $\frac{a_n}{a_{n-1}}$.

A) $n-1$ B) 5^n C) $5n$ D) $n!$

4. Given an arithmetic sequence (a_n) with $a_6+a_9=30$, find S_{14}.

A) 180 B) 210 C) 420 D) 450

5. Given that $g(x)=\begin{cases} x^2+5x-6 & x>1 \\ 5 & x=1 \\ 6x-1 & x<1 \end{cases}$, find $\frac{g(2)+g(1)}{g(-2)}$.

A) -1.5 B) 2 C) -2 D) -1

6. If $f(x)+f(x+1)=2x+3$ and $f(3)=4$, find $f(499)$.

A) 500 B) 998 C) 1000 D) 498

7. Simplify $\cos 20^0 \times \cos 40^0 \times \cos 60^0 \times \cos 80^0$.

A) $\frac{1}{2}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{1}{16}$ D) $\frac{\sqrt{3}}{3}$

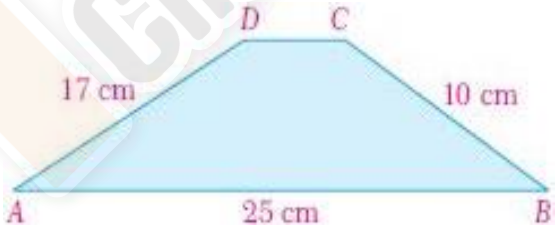
8. If $\frac{(298^2 - 98^2) - 200 \times 392}{2a} = 16$, find a.

A) 198 B) 25 C) 298 D) 45

9. If $mx^2 - 2(m-1)x + 3m - 1 = 0$ has two equal roots, then find the sum of possible value of m.

A) $-\frac{1}{2}$ B) $\frac{3}{2}$ C) $\frac{1}{2}$ D) 1

10. The 8 cm high trapezoid shown is rotated around its side AB. Find the volume of the solid of revolution created.



A) 448π B) 256π

C) 704π D) 653π

11. Find the sum of the roots of $x^{\log_3 3x} = 9$.

A) $\frac{25}{9}$ B) $3\frac{1}{9}$ C) $3\frac{5}{27}$ D) -1

12. If $f(x) = (3x+2)^{100}$, find $f^{(100)}(10)$ where $f^{(n)}(x)$ denotes the n^{th} derivative of $f(x)$.

A) $100! \times 51^{100}$ B) $100! \times 17^{100}$ C) $10! \times 3^{10}$ D) $100! \times 3^{100}$

13. If $a = 2022$, calculate $b = |a^2 - a + 1| - |a^2 + 1| + 2a + 5$.

A) 2023 B) 2025 C) 1 D) 2027

14. Evaluate $\frac{\cos^4 x - \sin^4 x}{\cos^2 x - \sin^2 x} - 1$, if $cos^2x - sin^2x \neq 0$.

A) $\cos x$ B) $\sin x$ C) 1 D) 0

15. If $\tan A = 3$, then find the value of $\frac{\sin A - \cos A}{\sin A + \cos A}$.

A) 2 B) $\frac{1}{2}$ C) $\sin A$ D) $\tan A$

16. If $1^2 + 2^2 + 3^2 + ... + 25^2 = k$, then calculate $2^2 + 4^2 + 6^2 + ... + 50^2$.

A) k^2 B) $2k^2$ C) $4k$ D) $k^2 + 2k$

17. If $\frac{2a+b-c}{a} = \frac{2b+c-a}{b} = \frac{2c+a-b}{c} = k$, then find the value of k.

A) 1 B) 2 C) a D) b

18. If $2^a = 5^b = 100$ and $x = \frac{1}{a} + \frac{1}{b}$, find the value of x.

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) 3

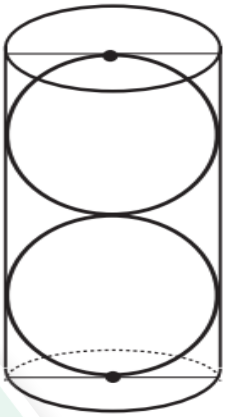
19. Find the remainder when 7^{2022} is divided by 8.

A) 1 B) 3 C) 5 D) 7

20. Evaluate $\int_1^e \left(x \ln x + x \ln^2 x \right) dx$.

A) 0 B) $\frac{e}{2}$ C) $\frac{e^2}{2}$ D) e^2

21. Two identical balls are fit in a cylinder. Find the ratio: $\frac{\text{Volume of two balls}}{\text{Volume of cylinder}}$



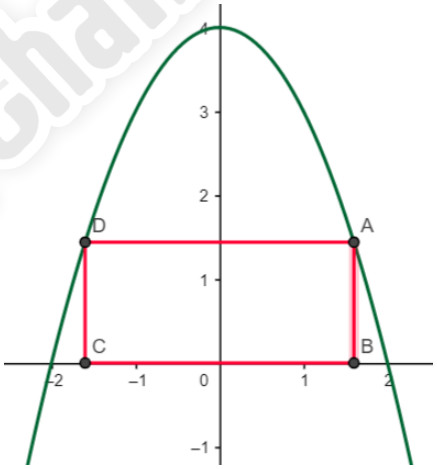
A) $\frac{3}{5}$ B) $\frac{1}{2}$

C) $\frac{1}{3}$ D) $\frac{2}{3}$

22. What number comes next 47592, 2574, 452, … .

A) 28 B) 25 C) 24 D) 52

23. A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 4 - x^2$. What is the height of the rectangle with the greatest possible area?



A) $\frac{4}{3}$ B) $\frac{8}{3}$

C) 2 D) 3

24. Solve the equation $\log_4 8x - \log_4(x-4) = 2$.

A) 6 B) 8 C) 9 D) 10

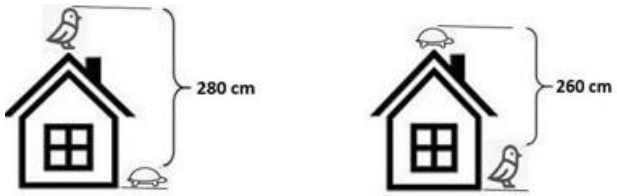
25. Which one of the following is a unit vector perpendicular to $\vec{A} = (5,12)$?

A) $(12,5)$ B) $\left(-\frac{12}{13},\frac{5}{13}\right)$ C) $\left(-\frac{12}{13},-\frac{5}{13}\right)$ D) $\left(-\frac{5}{13},-\frac{12}{13}\right)$

26. $\vec{V} = (1, a)$ is an angle bisector vector of two vectors namely, $\vec{V}_1 = (3,4)$ and $\vec{V}_2 = (12,5)$. Which of the following is a possible value for a ?

A) $\frac{5}{7}$ B) $\frac{7}{9}$ C) $\frac{9}{11}$ D) $\frac{11}{13}$

27. How tall is the house?



A) 230 B) 20 C) 540 D) 270 cm

28. Simplify $\frac{8^5 + 4^7 - 2^{13}}{2^{14} + 16^3}$.

A) 4 B) 8 C) 16 D) 2

29. Vladimir wants to text his friend from his mobile phone by using an old keypad given below and when he wants to insert "IMC", he codes it as follows: 444 6 222. According to the given coding system above, how can he input the word "WELCOME"?

1	2 ABC	3 DEF
4 GHI	5 JKL	6 MNO
7 PQRS	8 TUV	9 WXYZ
*	0 +	#

A) 4 1 1 7 7 7 2 2 2 0 0 0 6 4 4 4
B) 9 3 3 2 2 2 6 6 6 6 3 3
C) 9 3 3 5 5 5 2 2 2 6 6 6 6 3 3
D) 9 3 3 5 5 5 2 2 2 6 6 6 3 3

30. Evaluate $\sqrt{4 + \sqrt{9 - 4\sqrt{2}}}$.

A) $1 + \sqrt{2}$ B) $1 - \sqrt{2}$ C) $1 + \sqrt{3}$ D) $1 - \sqrt{3}$

31. Evaluate $\tan^2 45^0 - \cos^2 45^0 + \sin 30^0$.

A) $\frac{1}{4}$ B) $2\frac{3}{6}$ C) $\frac{5}{2}$ D) 1

32. If $2mx^2 + 20x + 5 = 0$ has equal roots ($x_1 = x_2$), find the value of m .

A) 3 B) 4 C) 7 D) 10

33. Evaluate $\frac{2}{\sqrt{49}+\sqrt{47}} + \frac{2}{\sqrt{47}+\sqrt{45}} + \frac{2}{\sqrt{45}+\sqrt{43}} + \ldots + \frac{2}{\sqrt{3}+\sqrt{1}}$.

A) $\frac{1}{7}$ B) 7 C) $5\sqrt{2}$ D) 6

34. If $P(x) = x^4 - 4x^2 + 5x - 2$, what is the constant term of $P(x + 2)$?

A) 2 B) 4 C) 6 D) 8

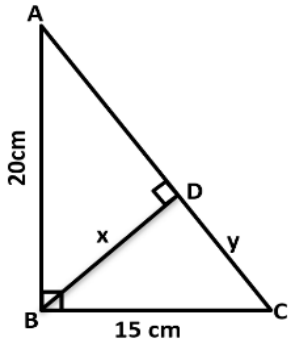
35. If $x^2 - 7 = \sqrt{7}x$, then find $x^2 + \frac{49}{x^2}$.

A) 9 B) 21 C) 7 D) 56

36. Given that $\cos 20^0 = \sin\left(\theta + 15^0\right)$, where $\theta + 15^0$ is acute angle, then find the value of θ.

A) 65^0 B) 55^0 C) 45^0 D) 35^0

37. Find the value of $x + y$.



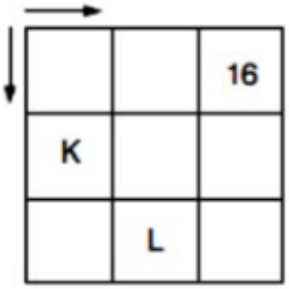
A) 21 B) 23

C) 25 D) 27

38. Calculate $2 + 15 \div 1 + 100 \div [17 + 16 \div (3 - 1)]$.

A) 21 B) 22 C) 23 D) 24

39. According to the following figure, each number in the square is doubled from left to right and halved from the top to down. Find the sum of K and L.



A) 4 B) 5

C) 6 D) 7

40. If $a-\frac{1}{a}=3\sqrt{2}$, then find the value of $\left(a+\frac{1}{a}\right)^2$.

A) 36 B) 22 C) 20 D) 18

Answers:

1. A) $\frac{101}{200}$ **2.** C) $\frac{27}{64}$ **3.** C) $5n$ **4.** B) 210 **5.** D) -1 **6.** A) 500 **7.** C) $\frac{1}{16}$ **8.** B) 25

9. A) $-\frac{1}{2}$ **10.** C) 704π **11.** B) $3\frac{1}{9}$ **12.** D) $100!\times 3^{100}$ **13.** D) 2027 **14.** D) 0

15. B) $\frac{1}{2}$ **16.** C) $4k$ **17.** B) 2 **18.** B) $\frac{1}{2}$ **19.** A) 1 **20.** C) $\frac{e^2}{2}$ **21.** D) $\frac{2}{3}$ **22.** C) 24

23. B) $\frac{8}{3}$ **24.** B) 8 **25.** B) $\left(-\frac{12}{13},\frac{5}{13}\right)$ **26.** B) $\frac{7}{9}$ **27.** D) 270 cm **28.** D) 2

29. C) 9 3 3 5 5 5 2 2 2 6 6 6 6 3 3 **30.** A) $1+\sqrt{2}$ **31.** D) 1 **32.** D) 10 **33.** D) 6

34. D) 8 **35.** B) 21 **36.** B) 55^0 **37.** A) 21 **38.** A) 21 **39.** A) 4 **40.** B) 22