A TRADITION OF EXCELLENCE

INSTRUCTIONS

You are about to take Copernicus Exam.

Please read the followings carefully.

1. The exam has 25 multiple choice-questions. Each question weighs 4 points. The maximum score a student can get is 100. There is a penalty of one point for each incorrect answer. So only answer the questions you are sure of.
2. Start with the easier questions, you can always come back to the questions you leave.
3. The time allocated for the exam is 60 minutes. You will start when the invigilator tells you to start.
4. You are required to comply with the directions given by the head invigilator before the examination.
5. Those who are taking the exam with a mobile phone MUST make sure that during the examination no one calls.
6. If anything in the examination is unclear, you can contact the invigilator.
7. Where permitted you may use a translation dictionary.
8. Students must not give or receive assistance of any kind during the exam. Any cheating, any attempt to cheat, assisting others to cheat, participating therein, or engaging in such improper conduct is a serious violation and will generally result in disqualifying.

Remember that “Hard work beats talent when talent doesn't work hard”

We wish you the very best luck on the exam.

1. How many zeros are there at the end of 720! ?

A) 150
B) 162
C) 178
D) 184

2. Find the remainder when 2022^{2022} is divided by 11.

A) 1
B) 4
C) 7
D) 9

3. Calculate the value of the expression below.

$$2 * \frac{\sqrt[3]{8^{n-2} + 7 * 8^{n-3}}}{\sqrt[4]{16^{n-1} - 16^{n-2}}}$$

A) 2
B) 5^{-12}
C) 3^{-12}
D) 15^{-12}

4. You throw three regular six-sided dice. What is the probability that you will get one odd number and two even numbers?

A) $\frac{1}{4}$
B) $\frac{3}{8}$
C) $\frac{4}{27}$
D) $\frac{1}{3}$

5. Points A, B, C, D, and E are on a line such that $AB = 3$, $BC = 6$, $CD = 8$, and $DE = 4$. What is the smallest possible value of AE?

A) 0
B) 1
C) 2
D) 3

6. Find $x + y + z$ if x, y, and z are natural numbers and

$$\begin{cases} x^3 - y^3 - z^3 = 3xyz \\ x^2 = 2(y + z) \end{cases}.$$

A) 0
B) 1
C) 2
D) 4

7. There are 20 students in a class. If one new boy joins the class, there will be twice as many boys as girls in the class. What is the product of the number of boys and the number of girls in the class?

A) 75
B) 84
C) 91
D) 96

8. If a, b, and c are natural numbers, how many pairs of roots does the equation below have?

$$a^3 + b^3 + 4 = c^3$$

A) 0
B) 1
C) 3
D) Infinite

9. Let x, y, and z be real numbers such that $3x + y = 1$, $3y + z = \frac{1}{2}$, and $3z + x = -\frac{1}{2}$. What is the value of $x + y + z$?

A) 1

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) $\frac{1}{4}$

10. Which of the following is equal to $\frac{1+\sqrt{2}}{\sqrt{2}-1}$?

A) $1 + \sqrt{2}$

B) $3 + 2\sqrt{2}$

C) $3\sqrt{2}$

D) $2 + 2\sqrt{2}$

11. Given that $4^{63} - 1$ is divisible by 103, find the integer n such that $n^3 - 1$ is divisible by 103 and $46 < n < 103$.

A) 48

B) 56

C) 64

D) 68

12. Let $ABCD$ be a kite with $AB = AD = 3$ and $CB = CD = 7$. A circle ω is inscribed in $ABCD$ (so that ω is tangent to all four sides). Find the largest possible radius of ω.

A) $\frac{19}{10}$

B) 2

C) $\frac{21}{10}$

D) $\frac{11}{5}$

13. Simplify the expression below.

$$(4 + \sqrt{15})(\sqrt{6} - \sqrt{10})\sqrt{4 - \sqrt{15}}$$

A) -2

B) 1

C) $\sqrt{2}$

D) $\sqrt{5}$

14. Find the value of

$$\left(\frac{1}{x-\sqrt{y}} + \frac{1}{x+\sqrt{y}} - \frac{2\sqrt{y}}{x^2-y}\right) \cdot (x + \sqrt{y}).$$

A) 1

B) 2

C) 3

D) 4

15. Find $x - y$ if

$$\begin{cases} y^2 = x^3 - 3x^2 + 2x \\ x^2 = y^3 - 3y^2 + 2y \end{cases}.$$

A) 0

B) 0 or 1

C) 0 or 2

D) 1 or 2

16. How many ways are there to cover a 4×4 square with only 2×2 and 1×1 squares, if tiles cannot be cut, exceed the boundary of the big square, or overlap?

A) 39

B) 41

C) 43

D) 45

17. Calculate the value of

$$\left(a-\frac{1}{a}\right)\left(\frac{1}{a-1}-\frac{1}{a+1}-1\right)\cdot\frac{5a}{3-a^2}.$$

A) 2
B) 3
C) 4
D) 5

18. Find the maximal integer value of a, when the roots of the equation below have different signs.

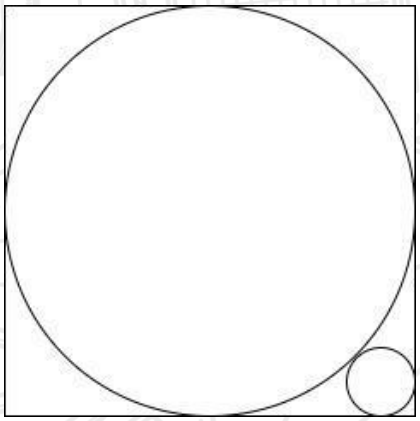
$$(a-2)x^2-3ax+a+5=0$$

A) -2
B) -1
C) 0
D) 1

19. Find the largest integer x such that $x^2+57x+2870$ is a perfect square.

A) 2022
B) 2025
C) 2027
D) 2029

20. The square in the figure has side length equal to 2. What is the radius of the small circle? (circles are touching)



A) $\sqrt{2}-1$
B) $\frac{2}{\sqrt{2}+1}$
C) $\sqrt{2}$
D) $3-2\sqrt{2}$

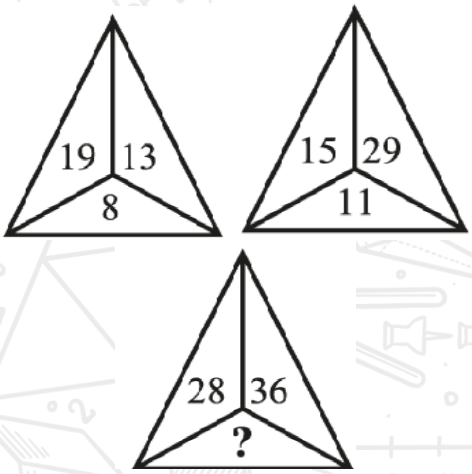
21. Find the simplest form of $\frac{\left(\sqrt{10}-1\right)^2-3}{\sqrt{10}+\sqrt{3}-1}$.

A) $\sqrt{10}-\sqrt{3}-1$
B) $\sqrt{10}+\sqrt{3}-1$
C) $\sqrt{7}-1$
D) $\sqrt{3}+1$

22. $10, 30, 32, 96, 98, 294, 296, ?$
What number should replace the question mark?

A) 888
B) 912
C) 818
D) 298

23. Find the missing number.



A) 12
B) 14
C) 16
D) 20

24. 473982 is to 1419 as 329684 is to 1418. Therefore, 751694 is to?

A) 1213
B) 1319
C) 1913
D) 2115

25. If $x + \frac{1}{x} = 4$, find the value of $x^3 + \frac{1}{x^3}$.

A) 8
B) 16
C) 8.5
D) 12