# 亞 洲 國 際 數 學 奧 林 匹 克 聯 合 會

**ASIA INTERNATIONAL MATHEMATICAL OLYMPIAD UNION**

# 亞洲國際數學奧林匹克公開賽初賽

**Asia International Mathematical Olympiad Open Trials**

# 高中組 Senior Secondary

時限：90 分鐘

Time allowed: 90 minutes

## 試題

## Question Paper

本試題不可取走。

THIS QUESTION PAPER CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

DO NOT turn over this Question Paper without approval of the examiner.

Otherwise, contestant may be DISQUALIFIED.

All answers should be written on the ANSWER SHEET.

Section A – each question carries 4 marks

---

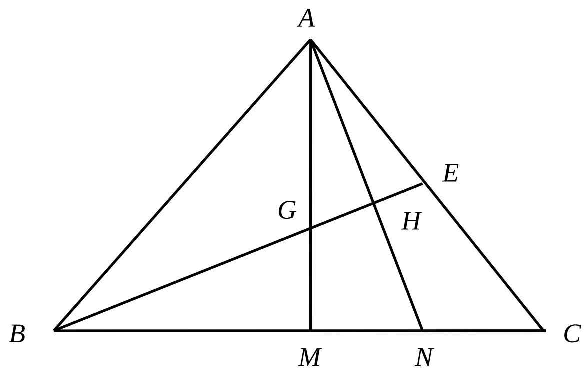
1) If $x$ is a 2-digit positive integer and $x^2 - 3y^2 = 1$. Find the largest possible integral value of $y$.

2) Given $[x]$ represent the largest integer less than $x$. Find the value of
$\left[\frac{7}{2016}\right]+\left[\frac{14}{2016}\right]+\left[\frac{21}{2016}\right]+\cdots+\left[\frac{8064}{2016}\right]$.

3) Express $4\sin 75^\circ - 2\cos 45^\circ$ in surd form.

4) If the determinant $\begin{vmatrix} 4x & 2x \\ 6 & x \end{vmatrix}$ has a value equal to $-9$, find the value of $x$.

5) In the figure below, $M$, $N$, and $E$ are the mid-points of $BC$, $MC$, and $AC$ respectively. Find $BG : GH : HE$.




6) Find the least possible positive integral solution $x$ to the congruence equation $x \equiv 4\times 5\times 6\times ...2014\times 2015\times 2016 \pmod{2017}$.

7) If $x$, $y$ are positive integers and follows the congruence equation $x^2 - y! \equiv 3 \pmod{10}$. Find the sum of all possible value(s) of $y$.

8) If $\omega = -\frac{1}{2} + i$, find the value of $\omega^4\left(\omega + 1 + \frac{1}{\omega}\right)^4$.

~ End of section A ~

All answers should be written on the ANSWER SHEET.

Section B – each question carries 5 marks

9) Given $[x]$ represent the largest integer less than *x*, find the product of all solutions to $7x-3[x]=4$.

10) In a rectangular coordinate system, there is a circle $x^2+y^2+2x-12y-4=0$ and a straight line. If the straight line passes through $A(5,2)$ and intersects the circle and point *B* and *C*. Find the value of $AB\times AC$.

11) Given *x* is a 3-digit positive integer and *x* follows the congruence equation $x^2-3x-28\equiv 0 \pmod{15}$, find the largest possible value of *x*.

12) Find the value of $\sum_{k=1}^{100}\left((-1)^{k+1}\times\frac{4k}{(2k)^2-1}\right)$.

13) It is known that *x*, *y*, *z* satisfy $\begin{cases} x+2y+3z=14 \\ 3x-2y+z=2 \\ 8x+5y-6z=0 \end{cases}$, find *xyz*.

14) It is known that (*a*, *b*) is the image when (3, 5) is reflected along $3x+4y+1=0$. Find the value of *a* + *b*.

15) If $a_n=5a_{n-1}-6a_{n-2}+2\times 3^{n-1}$, $a_1=11$ and $a_2=49$. Find the value of $a_{10}$.

16) If $f(x)=\frac{x}{\sqrt[3]{x^3+1}}$, find the value of $f^{91}(2)$.

~ End of section B ~

All answers should be written on the ANSWER SHEET.

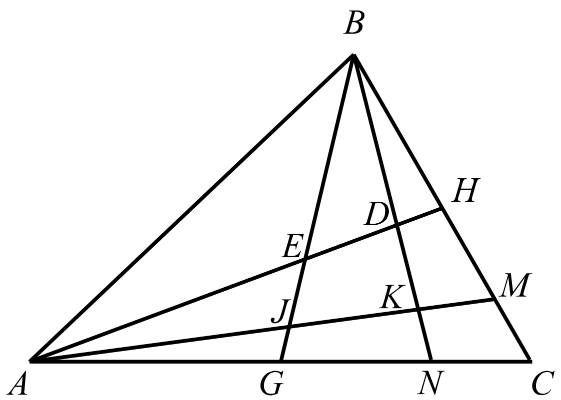
Section C – each question carries 7 marks

17) If $p = 1 - ai$ ($a \in \mathbb{R}$), $q = 3 - i$ ,and $p$ and $q$ are the solutions to the quadratic equation $z^2 - 4z + b = 0$ (b € R). Find the value of $ab$.

18) Let $f(n) = 5n^4 - 10n^3 + 10n^2 - 5n + 1$, find the last two digits of $\sum_{i=1}^{2016} f(i)$.

19) If $a_n = 7a_{n-1} - 10a_{n-2}$, $a_1 = 16, a_2 = 62$. Find the remainder when $a_{2016}$ is divided by 7.

20) In the figure below, $AG : GN : NC = BH : HM : MC = 5 : 3 : 2$. If the area of $\Delta ABC$ is 2016. Find the area of quadrilateral $DEJK$.




~ End of Paper ~