

亞洲國際數學奧林匹克聯合會

ASIA INTERNATIONAL MATHEMATICAL OLYMPIAD UNION



亞洲國際數學奧林匹克公開賽初賽

Asia International Mathematical Olympiad Open Trials

中三組 Grade 9

時限：70 分鐘

Time allowed: 70 minutes

試題

Question Paper

本試題不可取走。

THIS QUESTION PAPER CANNOT BE TAKEN AWAY.

未得監考官同意，切勿翻閱試題，否則參賽者將有可能被取消資格。

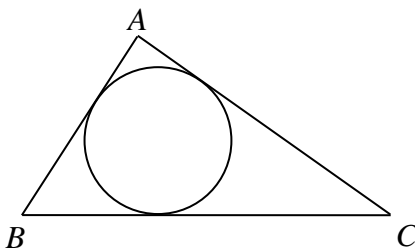
DO NOT turn over this Question Paper without approval of the examiner.

Otherwise, contestant may be DISQUALIFIED.

All answers should be written on the ANSWER SHEET.

Section A – each question carries 4 marks

- 1) If $x^2 + 9x + 20 = 0$, find the largest real possible value of x .
- 2) If $4x^2 \leq 8x - 4$, find the number of integral possible value(s) of x .
- 3) Find the remainder of $2015^{2017} \div 2016$.
- 4) If a straight line L passes through $A(-3, 9)$, and the slope of L is 4. Find the y-intercept of L .
- 5) If $\sin A + \cos A = \frac{1}{3}$, find the value of $\sin A \cos A$.
- 6) In the figure below, a circle is inscribed in $\triangle ABC$, if $AB = 30$, $AC = 40$ and $\angle BAC = 90^\circ$, find the radius of the inscribed circle.



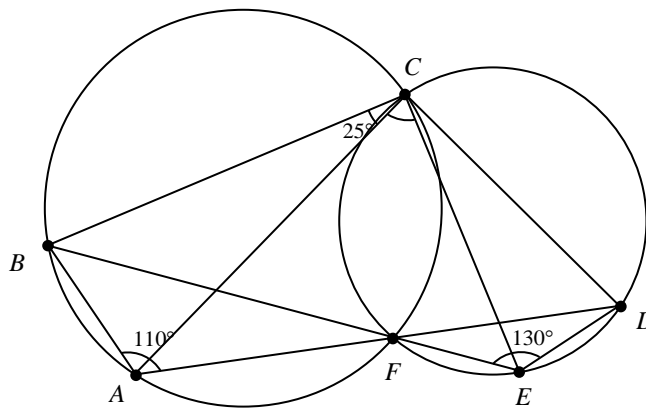
- 7) Evaluate $\sum_{k=0}^6 (kC_k^6)$.
- 8) Three fair six-sided dices are thrown, find the probability that the sum of the outcomes is even. (Show your answer in fraction)

~ End of section A ~

All answers should be written on the ANSWER SHEET.

Section B – each question carries 5 marks

- 9) If quadratic equation $9x^2 + 6x + (k - 7) = 0$ has real roots, find the largest possible value of k .
- 10) It is known that $|x|$ is the absolute value of x . If $x^2 - 3|x| + 2 \leq 0$, find the smallest possible value of x .
- 11) The three vertices of a triangle $\triangle ABC$ are respectively $A(-2,1)$, $B(3,-1)$ and $C(5,1)$. Find equation of the median of the triangle $\triangle ABC$ passing through A . Show your answer in general form.
- 12) In $\triangle ABC$, $BC = 12$, $AC = 11$ and $AB = 17$. Find the value of $\cos \angle A$. (Show your answer as fraction).
- 13) In the figure below, AFD and BFE are both straight lines. If $\angle BCA = 25^\circ$, $\angle BAD = 110^\circ$ and $\angle DEF = 130^\circ$ find the value of $\angle ACE$.



- 14) There are m red marbles and n white marbles in a hat. If three marbles are randomly drawn from it and the probability of drawing three white marbles is $\frac{1}{28}$, find the least possible value of n .
- 15) Find the largest possible value of $\sqrt{x-1008} + \sqrt{3024-x}$ for any real values of x . Show your answer in simplest surd.
- 16) If all of x_1, x_2, x_3, x_4, x_5 are not greater than 5, find the number of solution sets to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 15$.

~ End of section B ~

All answers should be written on the ANSWER SHEET.

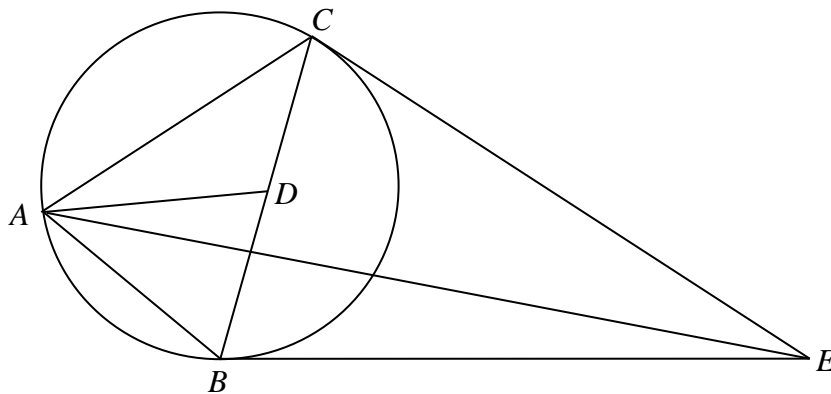
Section C – each question carries 7 marks

17) If x follows the congruence relations below, find the least possible positive integral value of x .

$$\begin{cases} 6x \equiv 5 \pmod{7} \\ 5x \equiv 3 \pmod{11} \\ 2x \equiv 9 \pmod{13} \end{cases}$$

18) If $x < 1$ and $(\log_7 x)^2 - \log_{49} x^8 = 5$, find the value of x .

19) In the figure below, $\triangle ABC$ is an acute-angle triangle and D is the mid-point of BC . A circumcircle of $\triangle ABC$ is constructed and tangents of the circle CE and BE is drawn, both lines meeting at E . If $\angle CAD = 30^\circ$, find the value of $2\cos \angle BAE$ and express your answer in surd form.



20) $f(x) = ax^3 + bx^2 + cx + d$ is a polynomial of highest degree of x not larger than three. If $f(1) = -1$, $f(2) = 5$, $f(3) = 29$, $f(4) = 83$, evaluate $a^4 + b^3 + c^2 + d$.

~ End of Paper ~