亞洲國際數學奧林匹克聯合會

ASIA INTERNATIONAL MATHEMATICAL OLYMPIAD UNION

亞洲國際數學奧林匹克公開賽初賽

Asia International Mathematical Olympiad Open Trials

中二組 Grade 8

時限：70 分鐘

Time allowed: 70 minutes

試題
Question Paper

All answers should be written on the ANSWER SHEET.

Section A – each question carries 4 marks

1) Find the value of r in the system of equations.

$$\begin{cases} r+s+t+u+3v = 8 \\ r+s+t+3u+v = 9 \\ r+s+3t+u+v = 7 \\ r+3s+t+u+v = 6 \\ 3r+s+t+u+v = 5 \end{cases}$$

2) If the product of a positive odd number and a positive even number is 2016, find the difference between the largest and the smallest possible value of the odd number.

3) If x and y are both integers, $x, y \geq 0$ and $2x + y = 100$. Find the number of possible values of y.
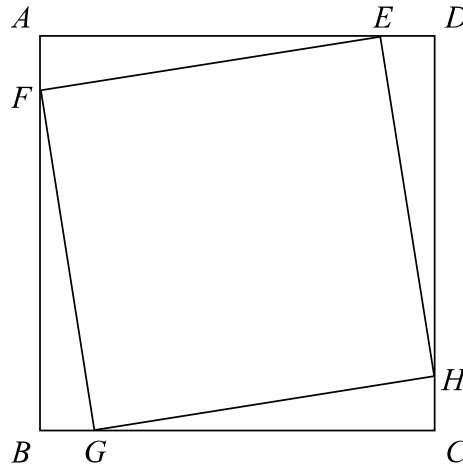
4) If the sum of the first n natural numbers is 2016, find the value of n.

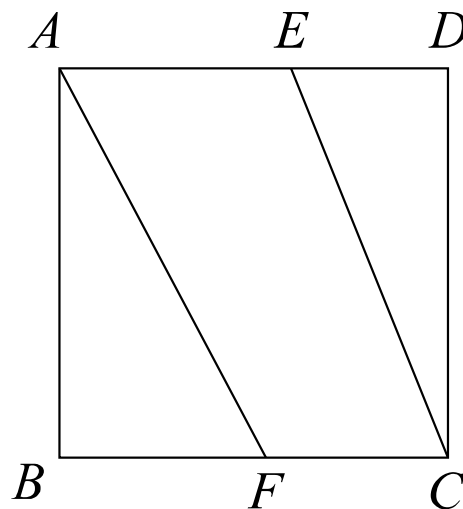5) Simplify $\dfrac{102487}{109648}$ to simplest fraction.

6) Find the integer nearest to $10\sqrt{\dfrac{6}{7}}$.

All answers should be written on the ANSWER SHEET.

7) In the figure, *ABCD* and *EFGH* both squares. If $AF : FB = 1 : 14$ and S_{ABCD} and S_{EFGH} denotes the areas of *ABCD* and *EFGH* respectively. Find $\dfrac{S_{ABCD}}{S_{EFGH}}$.

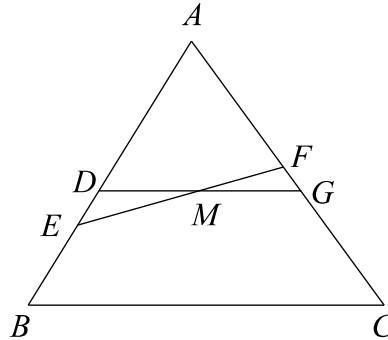8) In the figure, *ABCD* is a square and $BF + ED = DC = 4$, find the area of quadrilateral *AECF*.


~ End of section A ~

All answers should be written on the ANSWER SHEET.
Section B – each question carries 5 marks

9) In the figure below, $\angle AEF = \angle ACB$ and D and G are the mid-points of AB and AC respectively. If $AF = 6$, $AB = 15$ and $AC = 20$, find the length of DE.
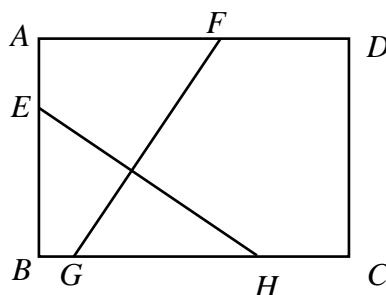

10) In the figure, $\angle AEF = \angle ACB$ and D and G are the mid-points of AB and AC respectively. If $AF = 6$, $AB = 15$ and $AC = 20$, find $S_{\Delta FGM} - S_{\Delta DEM}$. ($S_{\Delta ABC}$ denotes the area of ΔABC)
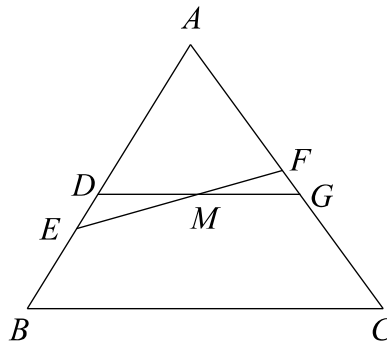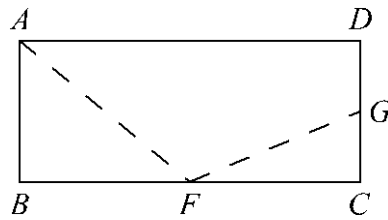

11) If $x, y \geq 0$ and $2x + 7y = 2018$, find the sum of all possible integral values of x.

12) In the figure below, $ABCD$ is a rectangle.
If $HE = FG$, $BH = AB$ and $\angle EHB = 35°$, find the size of $\angle AFG$.

All answers should be written on the ANSWER SHEET.

13) In the figure, *ABCD* is a rectangle and *F* is a point of *BC*. If $AD = 8$, $AD : AB : GC = 4 : 2 : 1$, find the minimum possible value of *AF + FG*.
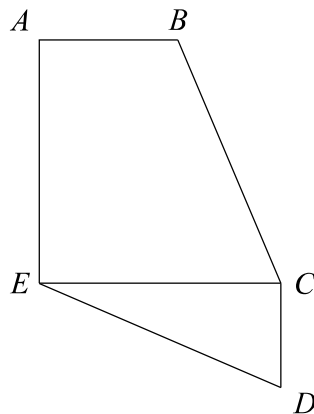

14) If $\begin{cases} x + \dfrac{1}{xy-1} = 7 \\ x - \dfrac{1}{xy-1} = 5 \end{cases}$, find the value of *xy*.

15) There are two numbers 17593 and 2993. From 1 to the lowest common multiple(L.C.M) of the two numbers (1 and the L.C.M. inclusive), there are *n* numbers that are divisible by the highest common factor (H.C.F.) of the two numbers, find the value of *n*.

16) In the figure below, $AB + CD = AE$, $\angle ABC = 105°$, $\angle CDE = 75°$, $AB \perp AE$, $BC = DE$ and $CE \perp CD$.

Find the value of $\dfrac{AC^2}{AE^2}$.

~ End of section B ~

All answers should be written on the ANSWER SHEET.

Section C – each question carries 7 marks

17) Let $a_1, a_2, \ldots, a_{21}$ denote the 21 integers from 1 to 21 in unknown order.
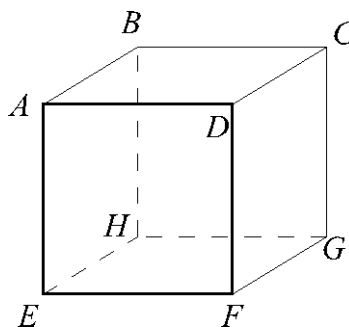If $k = (a_1 + 2001)(a_2 + 2002)\ldots(a_{21} + 2021)$, find the sum of the possible remainders when k is divided by 8.

18) If x, y are natural numbers, $y > 200$ and $3x^2 + xy - 2y^2 = 2000$. Find the value of x.

19) In the figure below, *EFD* and *AFC* are straight lines. If $BC = BF = AF$, $EC \perp BF$, $\angle FBC = \angle CAB$, find the value of $\angle DFC$.
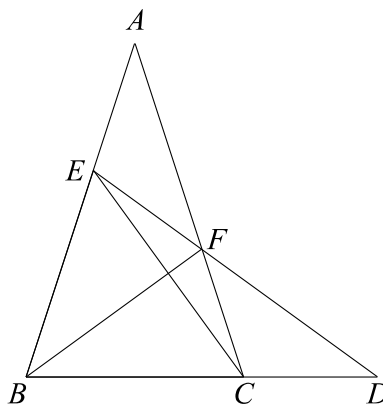
20) In the figure shows a cube with side length 20cm. A mosquito with negligible volume travels from the centre of the inner surface *ADEF*, it then touches the inner surfaces *ABCD*, *BCGH*, *EFGH* in order and ends at the centre of the inner surface *ADEF*. Find the shortest possible distance traveled by the mosquito and correct your answer to the nearest integer.

~ End of Paper ~